Assessment of Optimization Algorithms for Winglet Design

Design optimization of 3D winglets leads to performance improvement of a Piaggio Aero business jet

A numerical optimization procedure has been set up coupling a parametric CAD model, a structured mesh generator and a Navier-Stokes solver. The procedure has been applied to the design and optimization of 3D winglets of a business jet aircraft. Significant effort has been applied to the development of an efficient CAD model parameterization. Several preliminary optimization cycles have been carried out with the aim to define the optimal optimization strategy in terms of variable selection, constraints and target definitions. The final single objective optimization described in this article led to the successful design of a device and provided significant performance improvements, even with a strong constraint in the wing root bending moment. In a second phase, an optimization algorithm assessment has been performed on a typical 2D test case. The performances of SIMPLEX, genetic based and other advanced algorithms have been evaluated and compared in terms of quality of the optimum solution and convergence properties.

List of symbols
- Cd: 2D drag coefficient
- Cd: 3D drag coefficient
- Cl: 2D lift coefficient
- CL: 3D lift coefficient
- CMx: moment coefficient around X axis.
- CMx0: constraint limit of the moment coefficient around X axis.
- M: Mach number
- Obj.fn: objective function
- Re: Reynolds number
- α: angle of attack

Introduction

A design problem can be regarded as a creative process of searching for the "optimal" compromise solution within an iterative loop. The more complex and thorough the designer’s knowledge and experiences are, the higher the design quality will be. For this reason, considerable research has been done on how human designers work and in developing automatic decision making algorithms able to speed up such design processes [1]. Thanks to the improved reliability of modern numerical analysis tools and the exponential growth of computational power in recent years, numerical optimization methods are in great demand in nearly all industrial engineering areas. Today, numerical optimization is a leading design methodology in the aerospace sector, in both the industrial and research fields, and a key factor for competitiveness. Optimization is the process of finding a set of design parameters, known as design variables, x=(x1; x2;…;xn), that can be defined as optimal because they minimize or maximize one function f(x), called objective function, or a set of functions (mono-objective or multi-objective optimization). If the objective function (functions) has (have) to verify some constraints, the optimization is defined as constrained optimization.

The aim of this work is to describe an industrial application of an optimization process and to investigate the performance of several algorithms for a mono and multi-objective case.

The optimization of winglets suitable for a business aviation class aircraft is discussed in the first part of the article. In the second part, the results of the assessment of several optimization algorithms applied to the aerodynamic design of a 2D airfoil are detailed. In a mono-objective optimization
case, the SIMPLEX method developed by Nelder & Mead is compared to the performances of a Genetic Algorithm using a smart elitism operator. In a multi-objective optimization, four types of Genetic Algorithms and three Advanced Models are compared.

Part 1: Winglets optimization

The lift of an aircraft is generated by the pressure difference between the upper and the lower surface of the wing. The pressure difference induces a flow from the lower towards the upper surface around the leading edge and the wing tip [2]. This flow regime generates the so-called “tip vortices” which produce a down-wash effect. The down-wash effect is responsible for the local angle of the attach variation, and consequently for an additional drag component called “induced drag”. The induced drag is linked to the mechanism for lift production, its value is related to the strength of the tip vortex.

Winglets are wing tip devices whose objective is to recover part of the tip vortex energy in order to produce a force with a component in the forward direction. The effect is to generate some extra lift and to reduce the induced drag which decreases the strength of the vortex (similar effect as to increase the wing span).

The installation of well-designed winglets can improve the performance of an aircraft, however, the following aspects are critical:

1) The design must be strongly customized to each new configuration;
2) Winglets introduce additional weight;
3) They increase the wing root bending moment;
4) Efficiency is proportional to the lift coefficient;
5) They can alter the aerodynamics in critical regions (ailerons);
6) Winglets are expensive.

In this work, winglets for a business class aircraft have been designed and optimized in cruising speed conditions with the following constraints:
- The wing root bending moment increase should not be higher than 5%;
- No degradation of the wing characteristics (stall path and shock generation) is allowed.

The optimization procedure is schematized in figure 1. The objective function to minimize is the following:

\[ C_{\text{bend}} \leq C_{\text{bend,0}} \Rightarrow \text{Obj} = -\frac{C_{1}}{C_{2}} \]

\[ C_{\text{bend}} > C_{\text{bend,0}} \Rightarrow \text{Obj} = -\frac{C_{1}}{C_{2}} \left[ 1 - 20(C_{\text{bend}} - C_{\text{bend,0}})^{3} \right] \]

The constraint is introduced into the optimization function when the wing bending moment exceeds the limit value of \( C_{\text{bend,0}} \) which is the root bending moment of the wing without winglets increased by 5%.

Particular effort has been devoted to the development of an efficient and robust parametric CAD model. The objective was to generate a model able to reproduce the widest range of possible geometries with a minimum of parameters and to avoid the possibility to degenerate an unfeasible geometry under any parameter combination. The chosen topology is a blended winglet (figure 2) whose geometry is controlled by four parameters for the platform and two for the angle of incidence, listed as follows:

1) Cant parameter;
2) Taper ratio;

Fig.2 - Winglets CAD model and design parameters.

Fig.3 - Detail of mesh blocking topology
3) Winglet span;  
4) Sweep angle;  
5) Base angle;  
6) Tip angle.  

The airfoils at the tip and base section are also parameterized by a set of control points. However, preliminary optimization cycles showed that, in absence of shock or separation, their influence has a negligible effect on the optimization function. Therefore, the airfoils have been designed separately and frozen in the final optimization process we describe here.

The computational domain is automatically generated by a user defined script. It guides the code in the updating process of the geometry from the new CAD model. It projects the vertexes, the edges and the faces of a previous defined blocking topology to the updated geometric entities (figure 3); it also re-computes the structured hexahedral grid. Finally, it exports the new mesh in a format compatible with the CFD solver to be used. The grid has 2 millions of elements and the far-field is 30 chords.

The next step is the RANS computation on the new design. The Spalart-Allmaras turbulence model [3] has been used. In order to speed up the iteration, some simplifications have been adopted. The wall of the fuselage and the engine nacelle are considered inviscid. Wall function has been applied to the wing and winglets. The boundary conditions at the engine inlet (static pressure) and outlet (total pressure and temperature) are derived from the engine data.

The total elapsed time for one evaluation using 64 cpu on a Linux cluster, restarting the CFD run from the solution of the previous iteration, is about 10 minutes, 8 for the analysis and 2 for the geometry update and mesh generation (figure 4).

The optimization algorithm used is the SIMPLEX. The convergence of the solution was obtained after around 70 iterations. Figure 5 and 6 show the convergence histories of the six variables and the objective function. The aerodynamic efficiency was improved by 5% maintaining the wing root bending moment increase at 5.4%. The drag reduction of the completed aircraft was estimated at around 3% of the trimmed configuration.

The aim of the second part of this work is to assess some of the optimization algorithms of the modeFRONTIER optimization environment. The actual test case is the optimization of an airfoil in transonic conditions. The geometry is described by 28 design variables, which define the control points of a Bézier polynomial [4], [5]. The computational grid around the airfoil is automatically generated by a batch command. A coupled Euler/Boundary Layer 2D solver is used to evaluate the aerodynamic performance of the airfoil. Each evaluation requires about 20 seconds. Several mono and multi-objective optimizations have been performed.

Fig. 5 - Variables histories.
1) Mono-objective optimization using:
   a. SIMPLEX algorithm of Nelder & Mead (1965), with continuous and discrete variables;
   b. MOGA-II, Genetic Algorithm with a smart elitism operator [6];

2) Multi-objective optimization using:
   a. Genetic Algorithms:
      i. MOGA-II;
      ii. ARMOGA, based on the design range adaption;
      iii. FMOGA-II, which uses the response surface methodology;
      iv. NSGA-II, based on the Non-Dominated Sorting G.A. [7];
   b. Advanced Models:
      i. MOGT, based on the game theory (J.F. Nash, 1951), coupled with MOGA-II;
      ii. MOSA, based on Simulating Annealing [8];
      iii. MOPSO, based on Particle Swarm Optimization [9].

Mono-objective optimization
The objective function has been defined by weighting the drag coefficient in the following three design conditions.

1) \( \alpha = 2.8^\circ, \quad M = 0.734, \quad Re = 6.5 \cdot 10^6 \);

2) \( \alpha = 2.8^\circ, \quad M = 0.754, \quad Re = 6.2 \cdot 10^6 \);

3) \( \alpha = 1.8^\circ, \quad M = 0.68, \quad Re = 5.7 \cdot 10^6 \);

\[
\text{Obj} = 0.6 \cdot C_{d} + C_{p} + C_{n} \]

The reason for including more than one design point in very close flight conditions is to avoid the optimization process to converge to a shockless airfoil.

The starting solution was the RAE 2822 supercritical airfoil. The lift coefficient was always constrained while the minimum thickness constraint has been tested as an option leading to different solutions. Figure 7 details the convergence history of the optimizations without (7a) and with (7b) the imposition of the minimum thickness constraint.

Without the thickness constraint, the MOGA-II led to the best solution, despite the fact that it required about 30% more evaluations than the SIMPLEX. The SIMPLEX algorithm with discrete variables was the fastest to converge, but the optimum was not as good.

If the minimum thickness is a constraint, the SIMPLEX method proved to be more efficient than the MOGA-II, either by using discrete variables or continuous variables.

Multi-objective optimization
Two typical conflicting design targets have been selected to evaluate the multi-objective optimization algorithms:

1) Minimization of the drag coefficient in high-speed conditions:
   \( C_{d} = 0.8, \quad M = 0.734, \quad Re = 6.5 \cdot 10^6 \)

2) Reduction of the nose pressure peak coefficient in low-speed conditions at high incidence:
   \( \alpha = 10^\circ, \quad M = 0.2, \quad Re = 8 \cdot 10^6 \)

The minimum thickness of the airfoil was imposed as constraint. The Pareto fronts obtained by the several algorithms evaluated are compared in figure 8. In the same figure, a bar graph comparing the number of evaluations required to converge is reported.

All the models provided similar Pareto fronts. The ARMOSGA model seems to be a better compromise between solution and convergence velocity. The fastest algorithm is the hybrid MOGT coupled with MOGA-II, but in the present case, it did not properly investigate the entire solution space.

Conclusions
An efficient and robust optimization procedure has been set up coupling a parametric CAD model, a structured mesh generator and a Navier-Stokes solver. Winglets suitable for a business class aircraft have been designed and optimized in a complete aircraft configuration, including engines in flight. A single objective optimization using the SIMPLEX algorithm has been described. A drag reduction of 3% for the trimmed aircraft with the designed winglets installed has been estimated. In order to investigate the performance of other optimization configurations, an assessment of several algorithms applied to the mono and multi-objective optimization of a 2D transonic airfoil has been carried out. The performances of SIMPLEX, genetic based and other advanced algorithms have been evaluated and compared. In the unconstrained optimization, the SIMPLEX algorithm converged faster than the MOGA-II which, however, found a better solution exploring a wider range of variable combinations. The MOGA-II proved to suffer from the
thickness constraints imposition. In the multi-objective optimization, the several algorithms tested obtained very similar Pareto fronts. The ARMOGA seems to provide a better compromise between solution and convergence velocity. The hybrid MOGT coupled with MOGA-II is very fast but it failed to investigate the entire solutions space.

Future work
A detailed parametric study of algorithm performances requires huge efforts which could not be made in the frame of this work. Each algorithm might deliver a better performance than described in this article, if we tune it more accurately and orient it better to the specific case. Further activities are suggested to complete this investigation, in particular:
- to evaluate the efficiency of the algorithms with a deeper parametric investigation,
- to evaluate the sensitivity to the initial solution/population and select one or more test cases in order to be able to generalize the parametric investigation.

Bibliography