Multi-objective design optimization: application in roll-to-roll systems

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Outline

A. Introduction
   • Motivations
   • Experimental Plants

B. Plant modelling
   • Nonlinear and linear model
   • Model analysis

C. Decentralized control
   • Industrial Practice
   • Control optimization

D. Multi-objective control
   • Using modeFrontier

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A. Introduction

Longitudinal web dynamics

- In this presentation, we consider only longitudinal web dynamics. Longitudinal and out-of-plane vibrations are not taken into account.
A. Introduction

Roll-to-roll system: different elements

Unwinder

Master roll

Winder

dancer

Accumulator

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A. Introduction
Experimental Web Handling Platform at University of Strasbourg
B.1 Modelling of longitudinal web dynamics: nonlinear and linear model

⇒ simulator construction of longitudinal web dynamics
B.1 Modelling – nonlinear model

Web Tension Determination

- Hooke’s law

\[ T = ES\varepsilon = ES \frac{L - L_0}{L_0} \]

- Mass conservation law

\[ m = \rho SL = \rho_0 SL_0 \Rightarrow \frac{\rho}{\rho_0} = \frac{1}{1 + \varepsilon} \]

- Web tension between two consecutive rolls

\[ \frac{d}{dt} \left( \frac{L_k}{1 + \varepsilon_k} \right) = \frac{V_k}{1 + \varepsilon_{k-1}} - \frac{V_{k+1}}{1 + \varepsilon_k} \]
B.1 Modelling – nonlinear model

Web Speed Determination

- The velocity of the $k^{th}$ roll is obtained with a torque balance on it:

$$\frac{d}{dt}\left(\frac{J_k(t)V_k}{R_k(t)}\right) = R_k(t)\left(T_{k+1} - T_k\right) + C_m - C_f$$

  - Assuming no slippage, roll circumferential speed = web velocity

- Radius variations

$$\frac{dR_u}{dt} = -\frac{e}{2\pi} \frac{V_u}{R_u} = -\frac{e}{2\pi} \Omega_u$$

  and

$$\frac{dR_w}{dt} = -\frac{e}{2\pi} \frac{V_w}{R_w} = -\frac{e}{2\pi} \Omega_w$$

- Inertia

$$J_k = J_{0k} + \frac{\pi \rho l}{2} \left(R_k^4 - R_{0k}^4\right)$$
B.1 Modelling – nonlinear model

Pendulum Dancer Modelling

Diagram showing the pendulum dancer model with labeled parts:
- Cylinder
- Pivoting axis
- Roller
- Dancer arm

Mathematical equations and symbols are used to represent the dynamics of the system.
B.1 Simulator

Simulator in Matlab/Simulink environment

Construction of the multi motor simulator (Matlab®/Simulink®)
## B.2 Modelling – linear model

### State space variables of a 4-motor plant with 2 dancers

- **Use of linear model**: - system analysis (frequency domain)
  - linear controller synthesis

- **Example**: state space model of a 4-motor plant

\[
E(t) \frac{dX}{dt} = A(t)X + B(t)U
\]

\[
Y = C \ X
\]

\[
X^T = (V_1 \ T_1 \ V_2 \ T_2 \ V_3 \ T_3 \ V_4 \ T_4 \ V_5 \ \ldots \ldots)
\]
B.3 Model analysis

Influence of the Elasticity

- Example: transfer from the unwinder torque signal to the unwinder pendulum dancer position
  - 1600 MPa / 160 MPa / 16 MPa

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C. Decentralized control

Industrial control
C. Decentralized control

Industrial control practice: with load cell or dancer

With load cell

With dancer
C. Decentralized control

Industrial control scheme applied to our plant
Direct method for synthesizing optimized PID controllers:

- Theoretically difficult: we have to solve a BMI (non convex problem)
- Only mono-objective
C2. Fixed-order & -structure control

\( H_\infty \) fixed-order control

**LTI System \( \rightarrow N \) subsystems**:

\[
x = Ax + B_1 w + \sum_{i=1}^{N} B_{2i} u_i
\]

\[
z = C_1 x + D_{11} w + \sum_{i=1}^{N} D_{12i} u_i
\]

\[
y_i = C_{2i} x + D_{21i} w, \quad i = 1, 2, ..., N
\]

**Closed-loop form**:

\[
\dot{\tilde{x}} = (\tilde{A} + \tilde{B}_2 \tilde{K}_D \tilde{C}_2) \tilde{x} + (\tilde{B}_1 + \tilde{B}_2 \tilde{K}_D \tilde{D}_{21}) w
\]

\[
z = (\tilde{C}_1 + \tilde{D}_{12} \tilde{K}_D \tilde{C}_2) \tilde{x} + (\tilde{D}_{11} + \tilde{D}_{12} \tilde{K}_D \tilde{D}_{21}) w
\]

**Synthesis**:

\[
\left\| T_{wz} \right\|_\infty \leq \gamma \quad \text{BMI: with } \tilde{K}_D \text{ and } \tilde{P}
\]

\[
F(\tilde{K}_D, \tilde{P}) = \begin{bmatrix}
\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} & \tilde{P} \tilde{B}_1 & \tilde{C}_1^T \\
\tilde{B}_1^T \tilde{P} & -\gamma I & \tilde{D}_{11}^T \\
\tilde{C}_1 & \tilde{D}_{11} & -\gamma I
\end{bmatrix} + \begin{bmatrix}
\tilde{P} \tilde{B}_2 \\
0 \\
0
\end{bmatrix} \tilde{K}_D \begin{bmatrix}
\tilde{C}_2 & \tilde{D}_{21} & 0 \\
0 & \tilde{D}_{12}
\end{bmatrix} + \begin{bmatrix}
\tilde{P} \tilde{B}_2 \\
0 \\
0
\end{bmatrix} \tilde{K}_D \begin{bmatrix}
\tilde{C}_2 & \tilde{D}_{21} & 0 \\
0 & \tilde{D}_{12}
\end{bmatrix} < 0
\]

**Decentralized LTI Controller**:

\[
\dot{\bar{x}} = \begin{bmatrix} \bar{x}_1^T & \bar{x}_2^T & \ldots & \bar{x}_N^T \end{bmatrix}^T
\]

\[
\bar{A}_D = \text{diag} \{ \bar{A}_1, \bar{A}_2, \ldots, \bar{A}_N \}
\]

\[
\bar{B}_D = \text{diag} \{ \bar{B}_1, \bar{B}_2, \ldots, \bar{B}_N \}
\]

\[
\bar{C}_D = \text{diag} \{ \bar{C}_1, \bar{C}_2, \ldots, \bar{C}_N \}
\]

\[
\bar{D}_D = \text{diag} \{ \bar{D}_1, \bar{D}_2, \ldots, \bar{D}_N \}
\]

\[
\bar{K}_D = \begin{bmatrix} \bar{A}_D & \bar{B}_D \\
\bar{C}_D & \bar{D}_D
\end{bmatrix}
\]
D. Multi-objectives control optimization: using modeFrontier
Multi-objectives optimization

- Two ways to represent the dynamic behaviour of mechatronics systems
  - Time based approach
    - The system is simulated and its response is measured at each sampling period
    - Advantages:
      - Easy to implement
      - Physical correspondence of returned value (speed for example)
    - Drawbacks:
      - Results depends on the systems entries applied
Multi-objectives control design optimization

- Frequency based approach
  - We use the system transfer function in order to calculate its Bode diagram
  - Advantages:
    - This approach doesn’t depend on the system entries
    - We have a system behaviour overview for each frequency
  - Drawbacks:
    - Bode diagram not easy to interpret
    - System transfer function not always easy to calculate
    - Only applicable for linear systems
D Multi-objectives control design optimization

Application 1

Controller optimization of a 3-motor plant for a large elasticity range of flexible webs:

Web tension reference tracking:

\[ J_1 = \int_0^T (Tu_{ref}(t) - Tu(t))^2 dt + \int_0^T (Tw_{ref}(t) - Tw(t))^2 dt \]
D Multi-objectives control design optimization

Application 1

Web tension and velocity references:

Rewinder tension reference

Unwinder tension reference

Speed reference
D Multi-objectives control design optimization

Application 1

Workflow:

Performances:
- Reference tracking
- Velocity/tension decoupling

Sensitivity to web elasticity variations

DOE MOGA-II Matlab2

J2 J1

minimisation_2 minimisation

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D Multi-objectives control design optimization

Application 1

Performances:
- Reference tracking
- Velocity/tension decoupling

Sensitivity to web elasticity variations
D Multi-objectives control design optimization

Application 1

Web tension for a different web elasticity:
Multidisciplinary design optimization of an accumulator for a large elasticity range of flexible webs

Control strategy:
- C1 and C2
- controller switching law
D Multi-objectives control design optimization

Application 2

Controller optimization: 2 objectives
- Performances
- Sensitivity to web elasticity variations
Optimization of the winding tension based on the computation of the internal stresses in a wound roll:

- Roll obtained with optimized web tension
- Obtained with bad web tension
D Multi-objectives control design optimization

Application 3

Internal stress models:
- Wolfermann, 1976
- Connoly, Winarsky, 1984
- Bouquerel, Bourgin, 1993
- Olsen, 1996
- …

Winding tension reference?
Multi-objectives control design optimization

Application 3

Web tension optimisation (simplex algorithm) : $T_{\text{ref}} \ (R)$

Tang. Stress gauge
THANK YOU!