The use of race simulation models as an alternative to robust design optimisation

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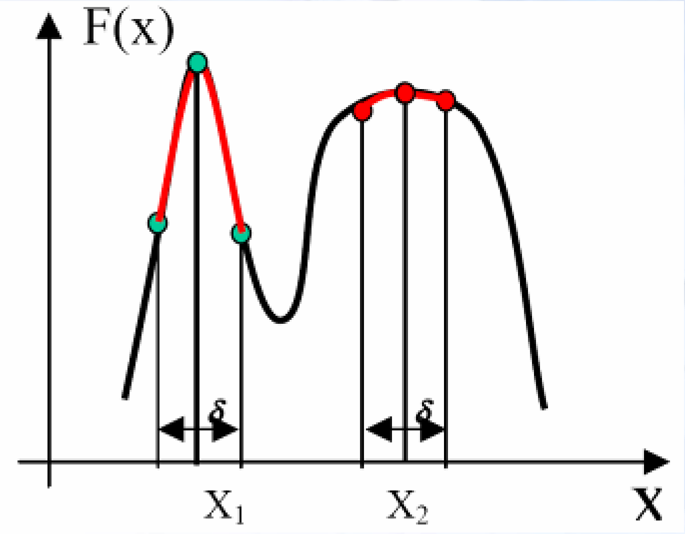
Outline

- Why do we need Robust Design Optimisation
- “Classical” ways of addressing Robust Design:
  - Weighted multi-point optimisation
  - Risk function
  - Mean value and variance (MORDO)
- Race Simulation methodology:
  - Idea
  - Example
  - Conclusions

Why do we need Robust Design Optimisation?

Uncertainty (*AIAA definition*): A potential deficiency in any phase or activity of the modelling process that is due to lack of knowledge:

- Uncertainties on **geometry parameters** due to manufacturing tolerance.
- Uncertainties on **operative conditions** (design point).

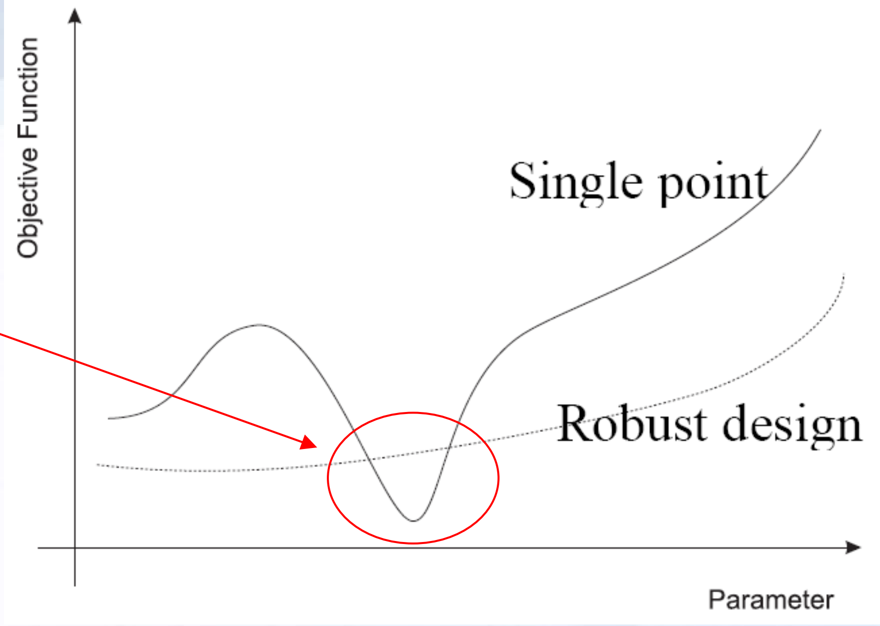


Why do we need Robust Design Optimisation?

Over-optimisation:

when optimising the objective function with fixed operative conditions, the final solution has usually good performance at the design point but poor off-design characteristics.

Usually the deterministic solution presents good performance only at the design point.



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- **“Classical” ways of addressing Robust Design:**
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Ways of addressing Robust Design Optimisation

- Weighted multi-point optimisation: the objective function is a weighted sum of different optimisations subjected to different operative conditions (to address the uncertainties)

$$\min \sum_{i=1}^{n} \omega_i \cdot c_d(d, Ma_i)$$

  Disadvantage: arbitrary definition of weights.

- Minimize a risk function ρ: $\min \rho = \int_{\mathrm{M}} c_d(d, Ma) \cdot f(Ma)\, dMa$

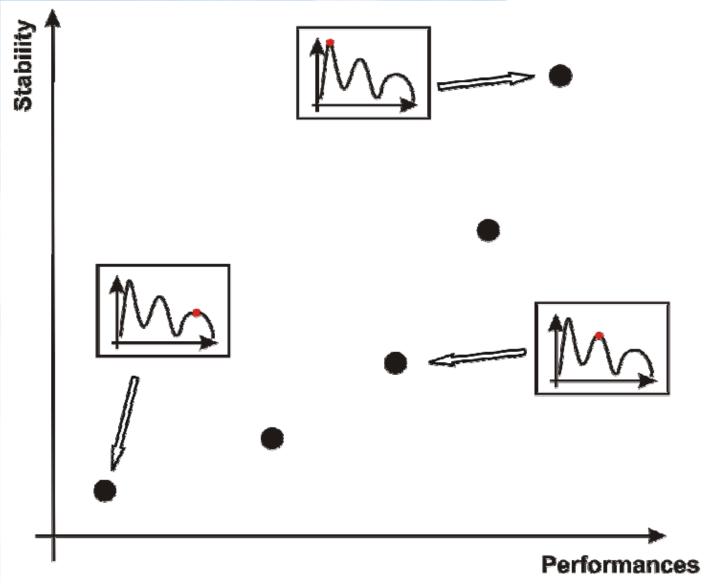
  Disadvantage: there is still the possibility of obtaining "unstable" solution.

Ways of addressing Robust Design Optimisation

- Optimise mean value of the objective function(s) and minimize its(their) standard deviation (***MORDO***):

$$\min E(c_d)\ ,\ \min \sigma(c_d)$$

- ***2* different directions** in the optimisation:
    - minimizing the variance of objective function, will minimize the off-design performance degradation;
    - optimising the objective mean value, the performances will be privileged.



Ways of addressing Robust Design Optimisation

Mean and variance values methodology (*MORDO*):

- Advantages:
  - No arbitrary definition of weights.
  - Set of solutions (Pareto front) from which to choose: high performance or stability of performance.
- But the Pareto front presents also some drawbacks:
  - Incrementing the number of objectives results in an increase of the number of evaluations.
  - Additional efforts are demanded in order to choose among the different designs.
  - If the uncertainties change slightly the Pareto front can still have the same behaviour.

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Race Simulation model

The idea comes from the *America's Cup*:

- The most challenging design is the one which wins most of the matches under stochastic perturbative conditions.
- The design which wins (maybe with a great gap) only one match under the most probable condition usually do not win the tournament.

This idea is directly applied to design under perturbative conditions (they are not part of the design itself: AoA, speed, etc.) but can be extended to uncertainties of the input parameters.

Race Simulation model

Race Simulation methodology:

- Binary matches in which each design competes against another design
- Each design will perform a number of matches in order to compete against each of the other designs
- For each match the conditions will be different.

Example of a tournament in a race simulation:

4 designs will result in 6 matches under 6 different conditions

A-B condition 1	B-C condition 4
A-C condition 2	B-D condition 5
A-D condition 3	C-D condition 6

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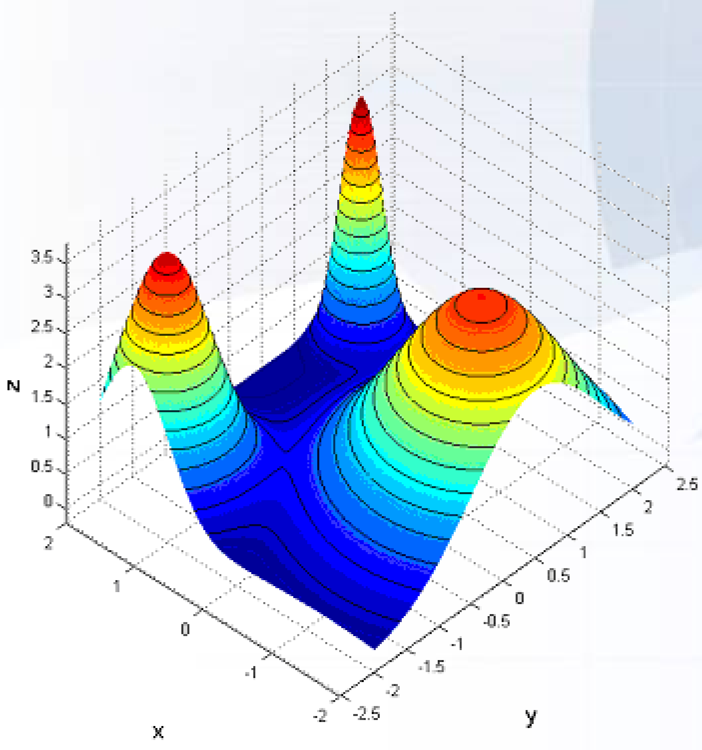
Race Simulation model

A numeric example: $$\max f(x,y) = \sum_{i=1}^{3} h_i e^{-\frac{a_i}{S_i}}$$ where:

$$s_1 = 0.1 \qquad h_1 = 3.5$$
$$s_2 = 0.5 \qquad h_2 = 3.25$$
$$s_3 = 2 \qquad h_3 = 3$$

$$a_1 = (x - 1.5)^2 + (y - 1.5)^2$$
$$a_2 = (x - 1.5)^2 + (y + 1.5)^2$$
$$a_3 = (x + 1.2)^2 + (x - 0.5)^2$$



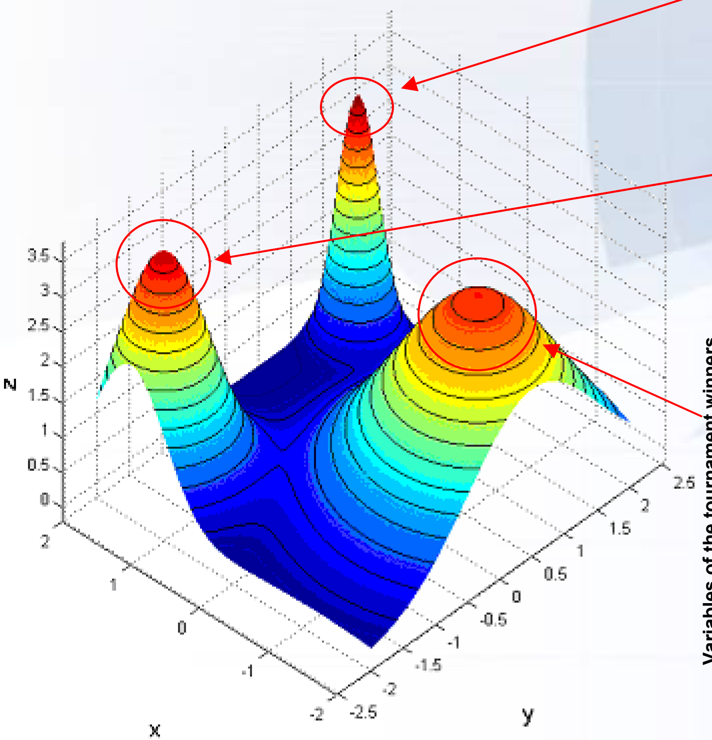
Three peaks with different behaviours

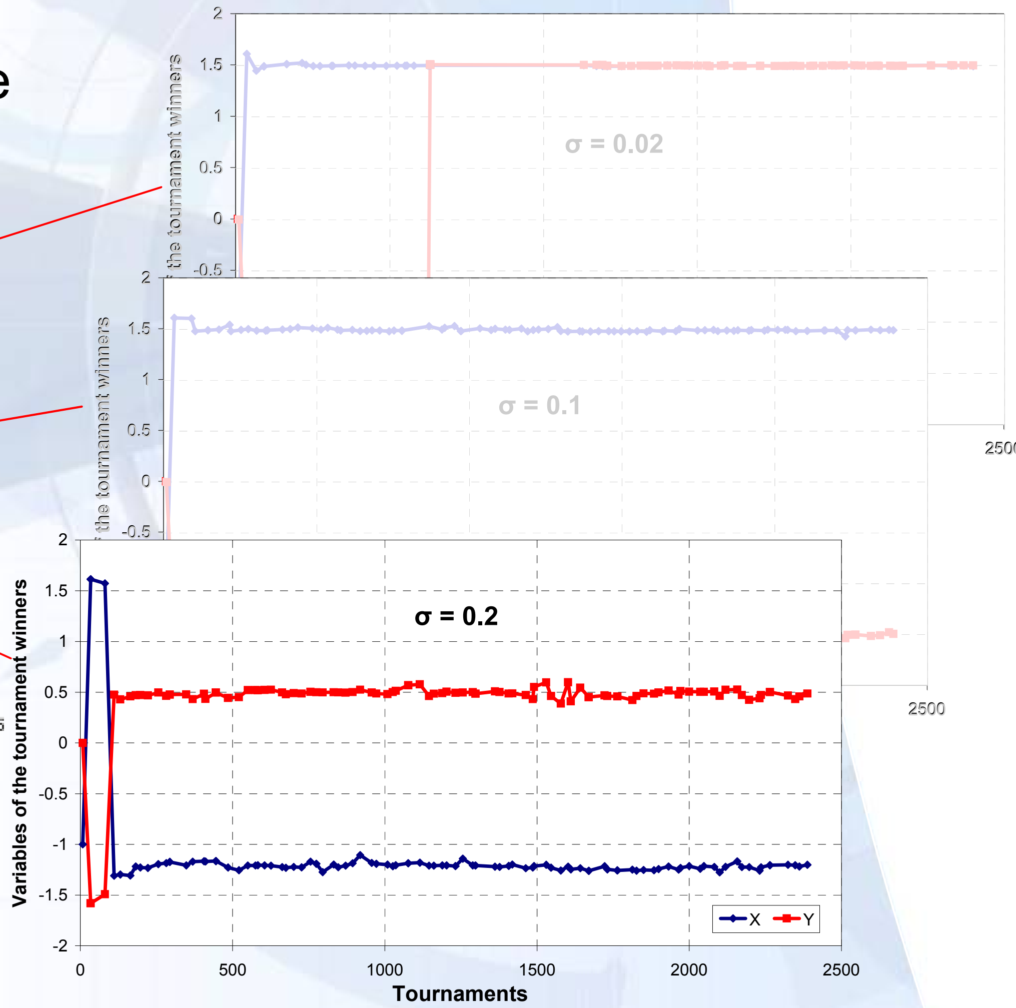
Uncertainty on the input parameters: X and Y defined inside the intervals $[x-\sigma_x, x+\sigma_x]$ and $[y-\sigma_y, y+\sigma_y]$

What happens with different values of the uncertainty σ?

Race Simulation model

The best design found by the race simulation depends on the perturbative condition.

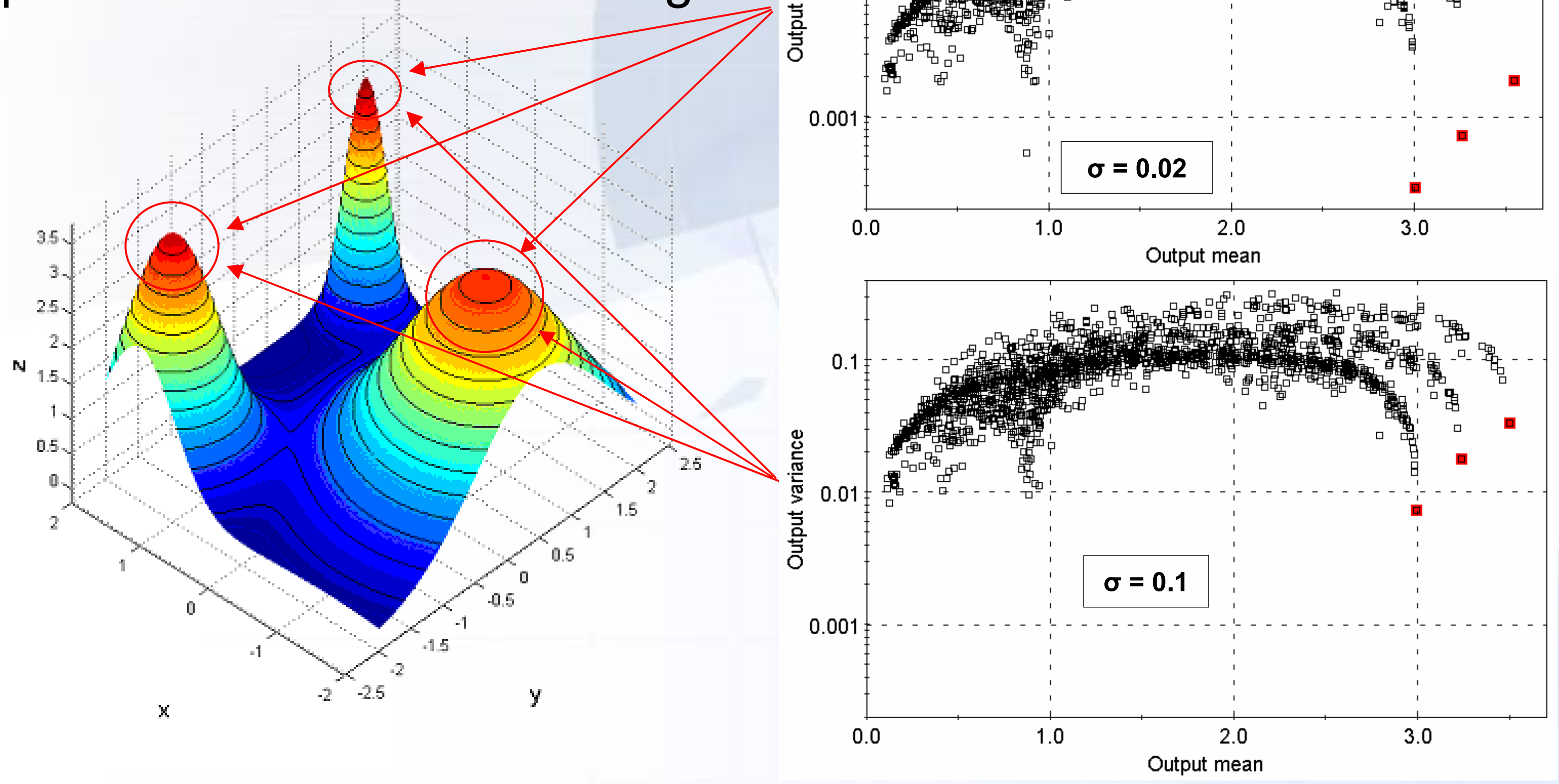




Race Simulation model

- Comparison with MORDO -

The Pareto front found with MORDO presents always the same behaviour also if the perturbative conditions change.



Race Simulation model

Race Tournament applied to an Airfoil test case:

- 5 different airfoil profiles with different performances.
- Perturbative condition (uniformly distributed): AoA.

Example preparation:

- RAE 2822 plus 4 optimised designs for 4 different AoA.
- Simulation of all of the different designs for a range of AoA, to be able to understand their behaviour and to compute mean and standard deviation.
- Tournament among these designs (each tournament comprise the simulation of each design for 4 different angles of attack).

Race Simulation model

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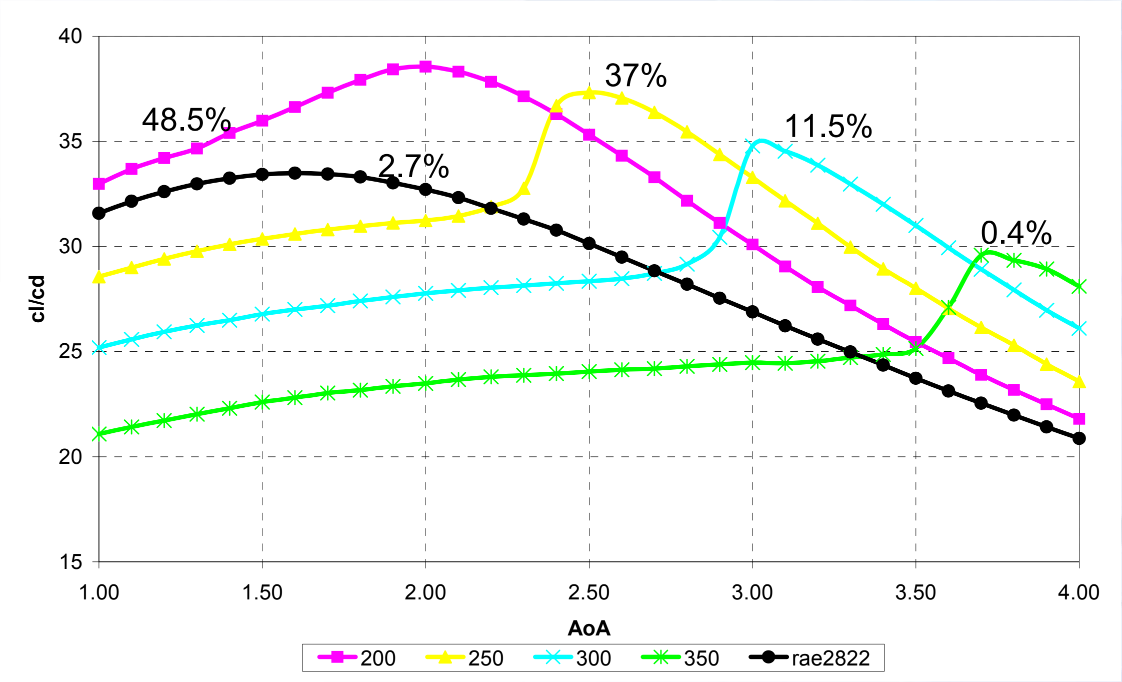
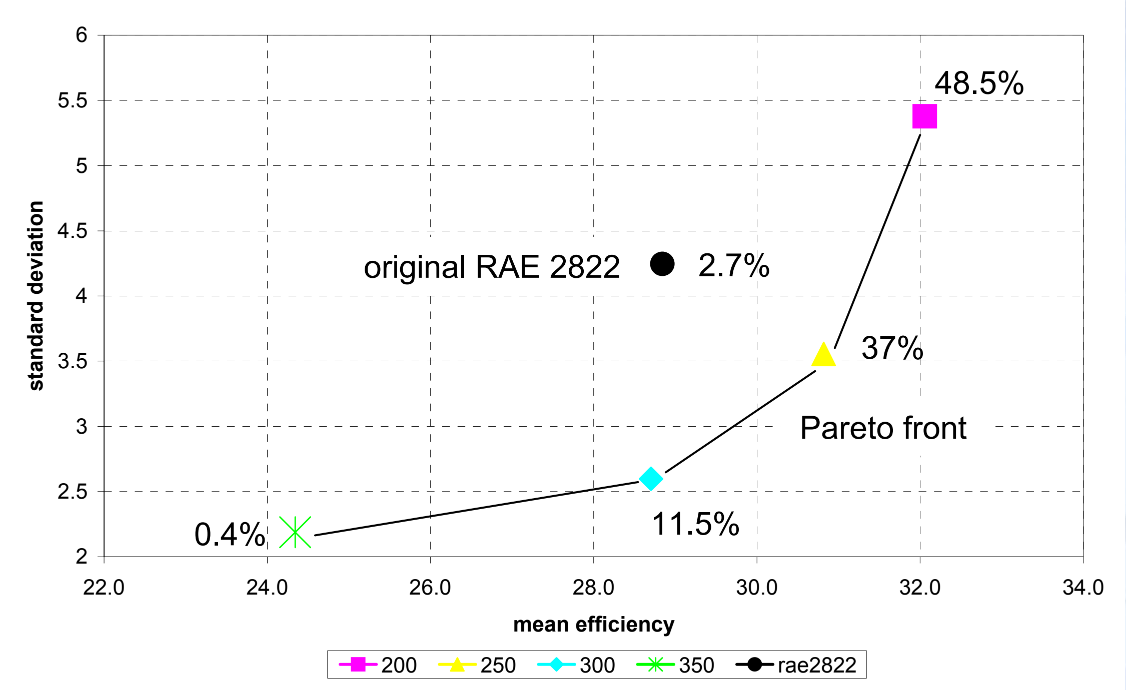


Race Simulation win probability:
2.00 ➔ 48.5%
2.50 ➔ 37.0%
3.00 ➔ 11.5%
3.50 ➔ 0.4%
RAE ➔ 2.7%

Race Simulation model

Mean value and variance ("classic" robust design values):

- The winner of the tournament is on the Pareto front.
- The worst design is on the Pareto front as well.



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Race Simulation model: conclusions

Advantages of Race Simulation methodology:

- No arbitrary weights definition.
- No increment of the objectives number.
- No preliminary preference for performance or stability: the result of the tournament is an *implicit weighted function* of mean and variance of performance.

Disadvantages:

- A tournament can be very time consuming (but not as MORDO):
  - reduction techniques of matches per tournament;
  - use of response surfaces.
- The tournament score can not be directly used as the overall optimisation objective.

Race Simulation model applications

How to apply the methodology to optimisation algorithms:

- MORDO with race tournament:
  - Evaluate the tournament for each generation.
  - Use the tournament score as fitness value to assign the probability of the selection operator.
  - Compute the mean and variance of the objective as in standard robust design.
- Race tournament:
  - Evaluate the tournament for each generation.
  - Use the tournament score as the objective function, without keeping memory of it in the following generation.
  - Always copy to the next generation the best design(s) of current generation.