# Chained Bezier Parameterization for Shape- and Bubble-Method Topology Optimization

Damir Vučina, Željan Lozina, Igor Pehnec

FESB, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split

R.Boskovica bb

21000 Split, Croatia

*vucina@fesb.hr*

# Scope

- Development of computational framework and suitable layout for shape optimization (optimum geometric design) and bubble-based topology synthesis
- Elements: standard existing programs as brick-stones (affordable in terms of costs of implementation, efficient in computing performance, no steep learning curve for the user) → ModeFRONTIER

# Framework for shape optimization

- The system must integrate:
  - geometric modeling tools (CAD),
  - evolutionary optimizers for multi-objective constrained non-linear hybrid problems with continuous and discrete variables,
  - simulators of candidate design’s performance, finite-element-analysis (FEA) programs,
  - synchronization and intensive exchange of data amongst these components.

# Simultaneous topology and shape optimization

Optimization **variables**: descriptors of topology and shape of domain,

with excellence **criteria** typically used: minimum mass and compliance

subject to **constraints**: equilibrium or minimum energy, given boundary conditions, sustaining the given loading, permissible levels of equivalent stresses within the domain, permissible displacements, etc.

$$\min f^T u \quad , \quad \min m \qquad \text{with} \qquad K(E) \cdot u = f$$

Two fields to be determined: E, u

# Material distribution approach /1

Based on subdivision of domain into small partial (cell-like) areas, where certain respective material-related properties (density, etc) are modeled using 0/1 binary variables

$$E = 1_{\Omega_{mat}} \cdot E^0 \qquad , \quad 1_{\Omega_{mat}} = \begin{cases} 1 & if\ x \in \Omega_{mat} \\ 0 & if\ x \in \Omega \setminus \Omega_{mat} \end{cases}$$

$$\int_{\Omega} 1_{\Omega_{mat}} \cdot d\Omega \leq V$$

Two field variable distributions to be determined: distribution of material (density) and distribution of displacements

# Material distribution approach /2

For numerical reasons binary variables replaced by continuous variables which tend towards 0 or 1, penalizing intermediate values

$$E(x) = \rho(x)^p \cdot E^0 \qquad , \quad p > 1$$

$$\int_\Omega \rho(x) d\Omega \leq V \qquad , \quad 0 \leq \rho(x) \leq 1 \qquad , \quad x \in \Omega$$

SIMP formulation converts the material distribution problem with binary variables into a sizing problem on a fixed discretized domain.

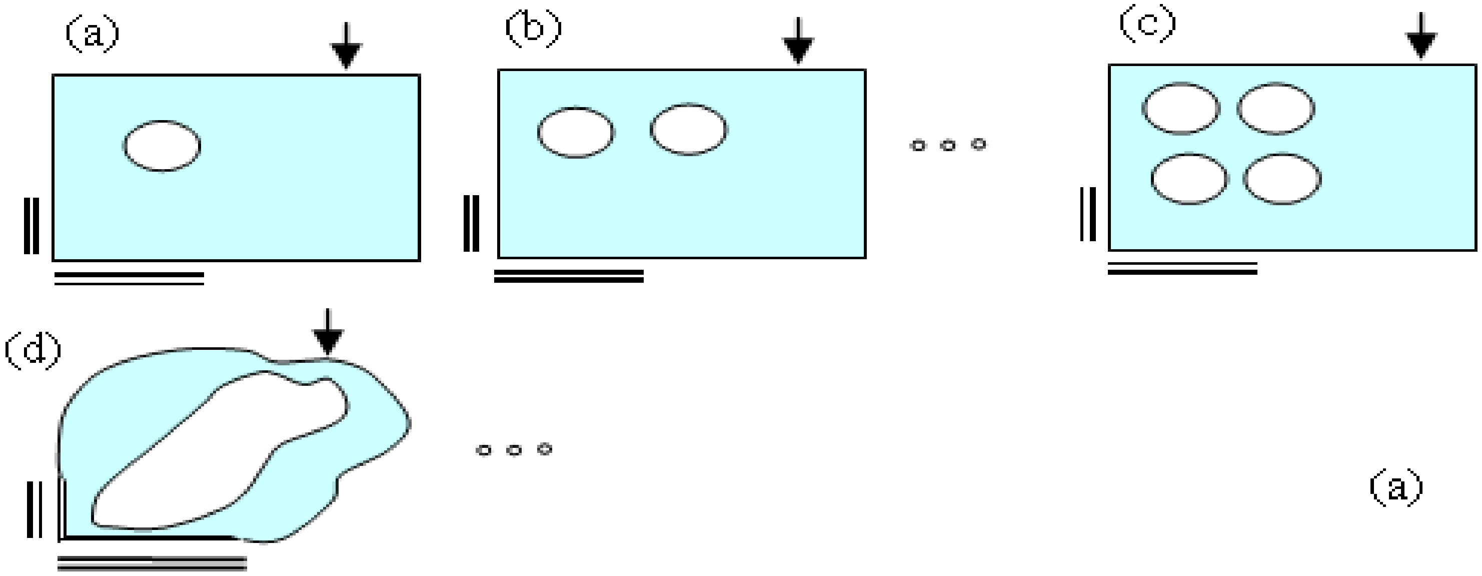
Resulting space (bit-map alike) filled with material has to be numerically processed and interpreted in terms of boundary curves or surfaces.

# Bubble approach for topology & shape /1

Different approach applied here: successively increasing the degree of the topology and solving the resulting shape optimization problem in each step.

Furthermore, the shape optimization problem is converted into the dimensional optimization problem as shapes are represented by parametric curves and surfaces. Hence possibility to apply parameter variation methods along with FE simulators, which yields an inexpensive optimum design infrastructure

# Bubble approach for topology & shape /2




Advancing horizontally from (a) to (c) illustrates topological optimization with an increasing topological degree of complexity, while advancing vertically from (a) to (d) illustrates shape optimization within the same topology. An approach based on a combination of both is proposed.

# Bubble approach for topology & shape /2

Basic implementations insert bubbles of predefined shape such as circles, ellipses, squares or triangles into the domain, hence fewer optimization variables (as only center positions, orientations, and corresponding basic dimensions are considered).

More general and flexible approach applied here; holes and outer contours have a general shape. The optimization variables are the polar coordinates $r(\varphi)$ of the free control points of individual contours which are then transformed into chained piecewise parametric contour curves.

Evolutionary optimizers are employed.

# Representation of shape, parameterization

Representation of shape to be employed should provide locality, sufficient representation flexibility, global representation capacity possibly with local focus, and numerical efficiency and stability.

<u>Parameterization</u>:
- Results in a compact set of shape parameters,
- Does not bias the search by having the optimizer implicitly inherit any search preferences,
- Behaves locally, isolated changes in the shape parameters only cause local changes in shape,
- Offers sufficient control in representing both global and local shape of objects.
- It is complete and capable of representing any variation of shape possible within the defined search space,
- Preserves a one-to-one type of relationship in the mapping between the geometric shape and shape parameters, otherwise different values of the parameters might be representing the same shape. This could have a serious impact on the search efficiency and convergence properties, or even more negatively, some possible shapes might not be representable at all,
- Ensures that small changes in the parameters should correspond to small changes in the shape represented (mapping), otherwise divergence may occur,
- Uses a set of parameters fixed in structure and size, which implies that the mapping is permanent for the entire search space, otherwise the shape optimizer would have to be adaptive,
- Enables easy communication of data between the elements of the workflow such as the optimizer, geometric modeler- CAD and FE simulator, which is important from the practical point of view concerning the corresponding numerical implementation,
- Provides for simple detection of invalid candidate designs occasionally generated by the evolutionary shape optimizer's operators of cross-over, mutation and other. The candidate designs can be infeasible in terms of respective topology, shape, dimensions, geometric interference with other objects, locking configurations, imposing of non-existing mobility constraints, etc,
- Provides for numerically efficient algorithm implementation.

# Bezier curves and surfaces

Approach: chained Bezier curves and surfaces

$$\mathbf{P}(u) = \sum_{i=0}^{n} B_{i,n}(u) \cdot \mathbf{P}_i \quad = \quad \sum_{i=0}^{n} \binom{n}{i} \cdot u^i \cdot (1-u)^{n-i} \cdot \mathbf{P}_i$$

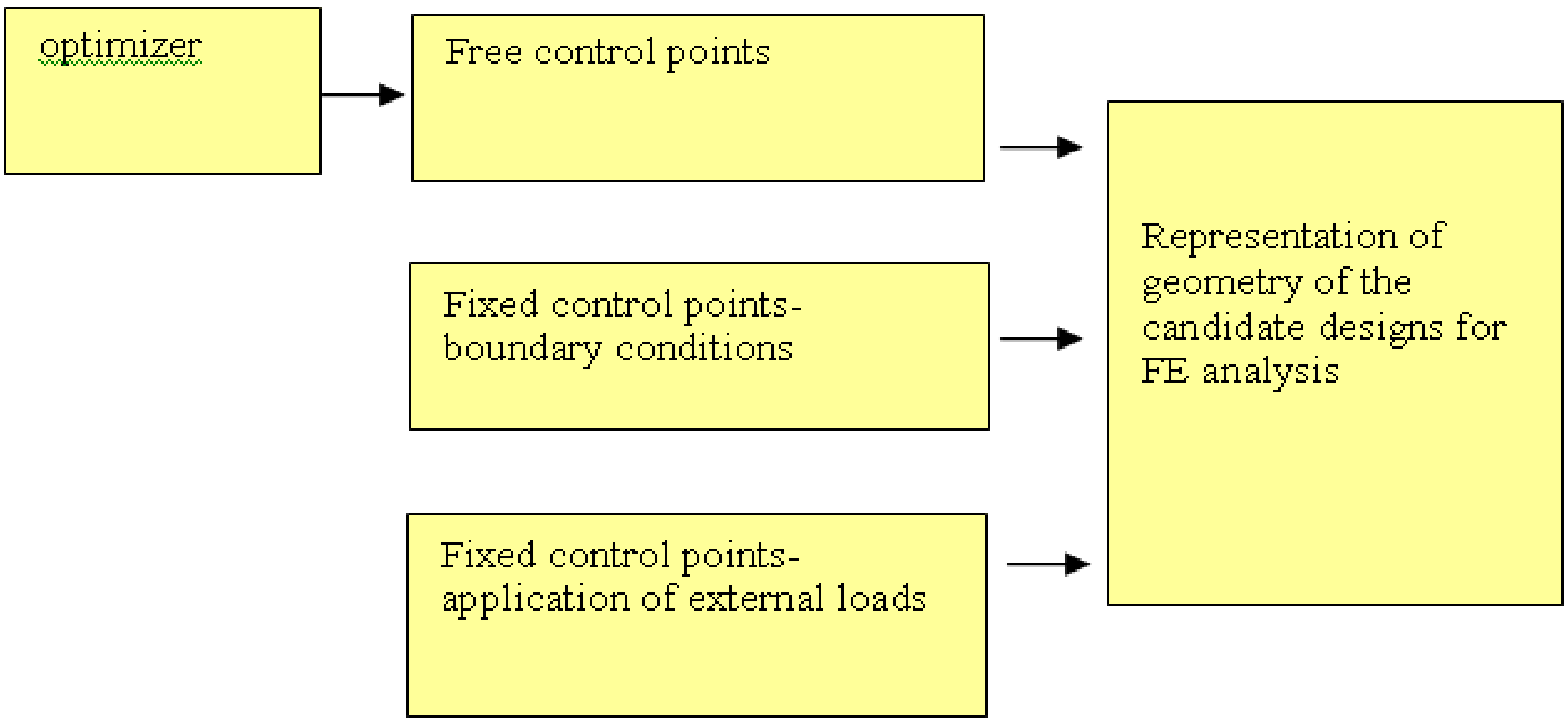
$$\frac{d\mathbf{P}(u)}{du} = n \cdot \sum_{i=0}^{n-1} \binom{n-1}{i} \cdot u^i \cdot (1-u)^{n-1-i} \cdot A_i \qquad , \quad A_i = P_{i+1} - P_i$$

$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,n}(u) \cdot B_{j,m}(v) \cdot \mathbf{P}_{i,j} \qquad , \quad u,v \in (0,1)$$

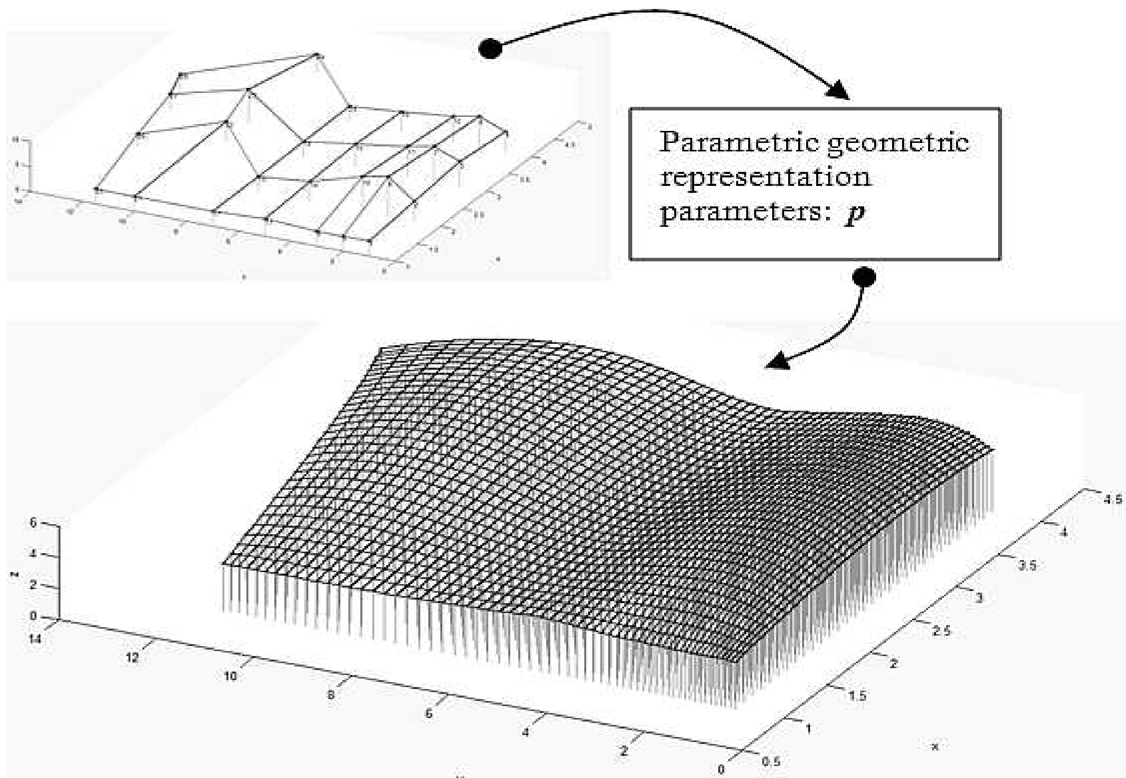
# Chaining of Bezier curves and surfaces

- curves of different degrees can be chained, small areas of particular interest can be modeled more accurately with more densely populated control points and combined with large monotonous areas with low density of control points
- locality is fully provided for
- high numerical efficiency due to locality and simple implementation of Bezier formulas and their chaining
- processing of the geometry (chained curves and surfaces) is inherently parallel, therefore implementation of parallel processing of the geometry is possible
- different degrees of continuity between chained curves are possible

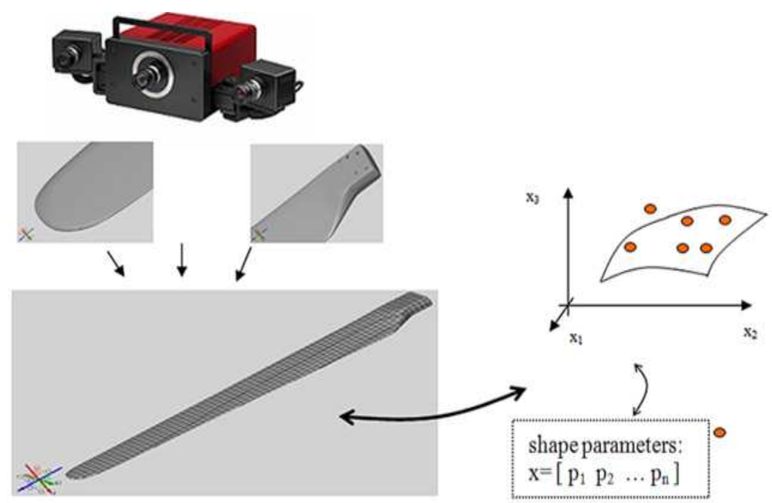
# Control points structure



# Parametric representation of shape



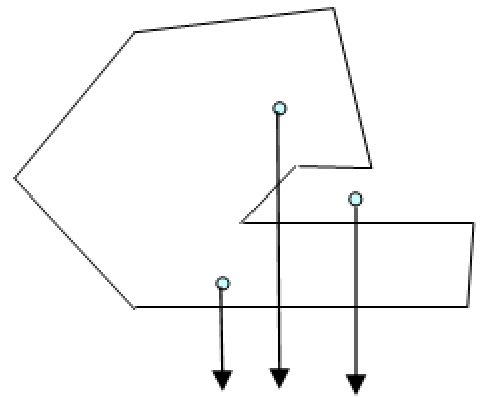
# Possible application: 3D scanning, reverse engineering, optimizaton



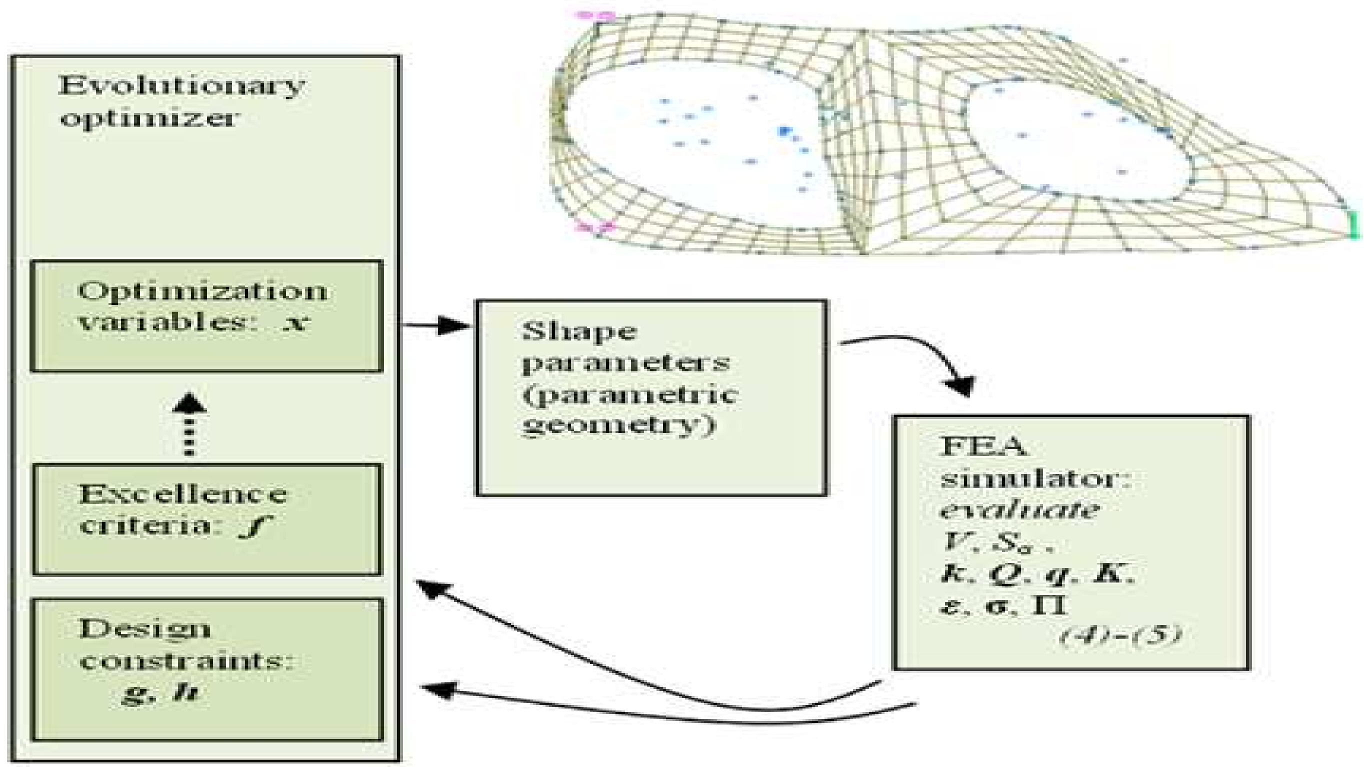
# Detecting topological relationships between objects in the domain

Jordan theorem: a closed polygon devides the space into two regions, interior and exterior, and a point is interior to the closed polygon if an arbitrary ray from the point crosses the polygon an odd number of times.

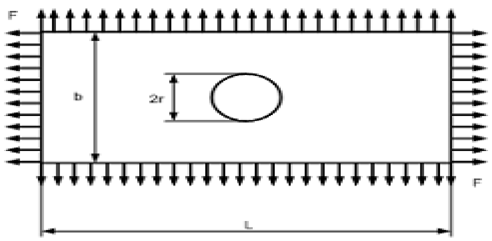
There are also similar approaches that can be used to test whether a 3D point is contained within a 3D object with triangulated mesh boundaries.

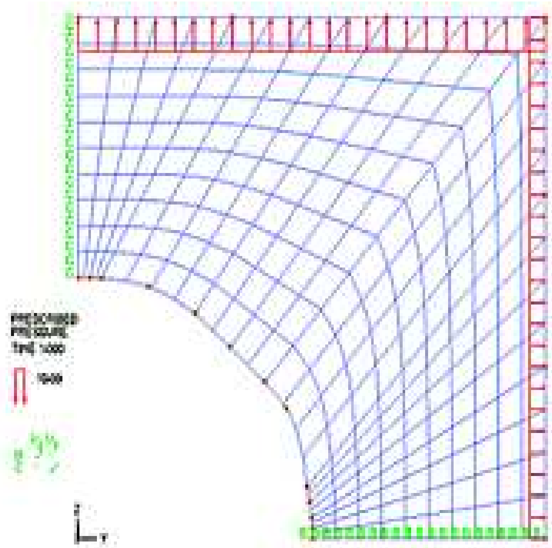


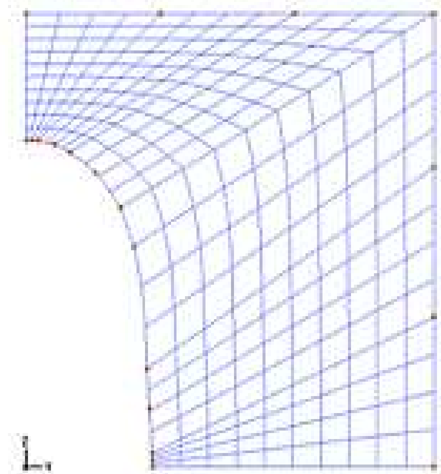
# Overall layout of workflow

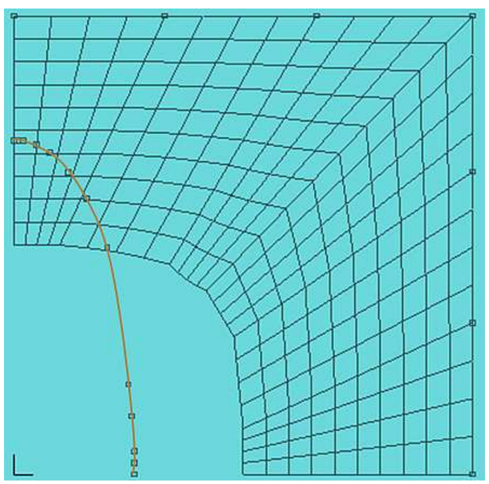


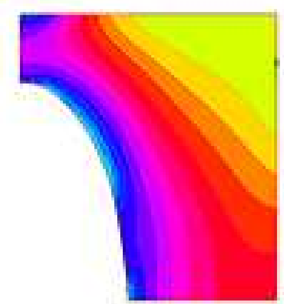
# Test example /1



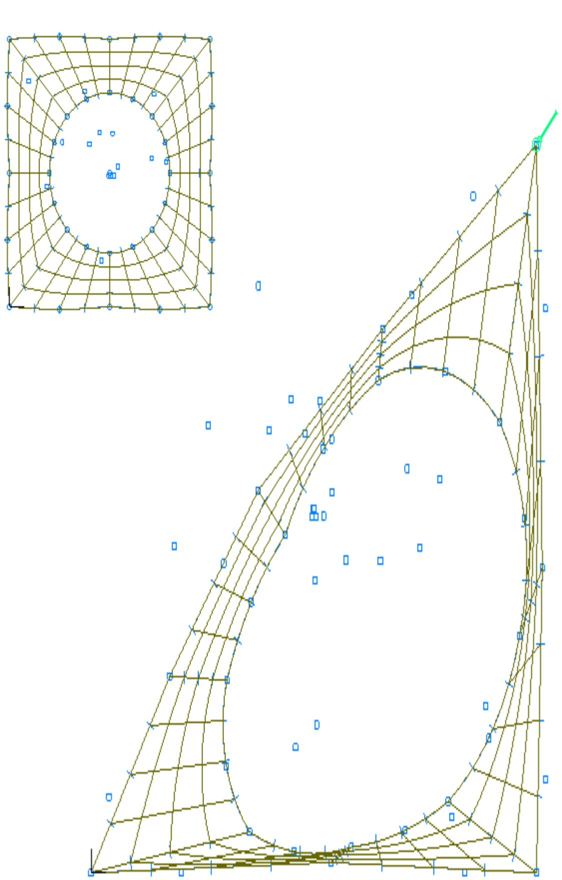



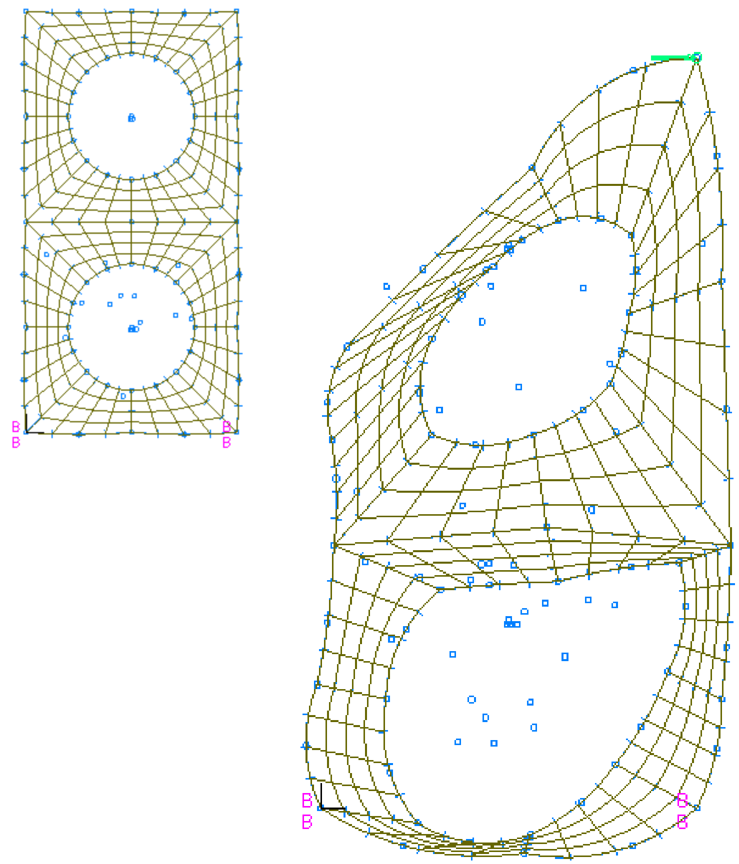






# Test example /2- transitions





Basic parameterization with
21 variables for 3 contours

# Conclusions

- Combining topology and shape optimization using the hole (bubble) approach
- Development of 2D and 3D shape representation based on chaining Bezier curves and surfaces
- Development of workflow to implement the procedure
- Ongoing work with more realistic examples