Stirling engine optimization based on Schmidt and adiabatic analyses

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Outline

1. Numerical analysis
    - Stirling engine
    - Schmidt analysis
    - Adiabatic analysis
2. Optimization technique
    - Problem definition
    - Optimization procedure
3. Results
    - Schmidt analysis
    - Adiabatic analysis
    - Schmidt vs adiabatic
    - Remarks

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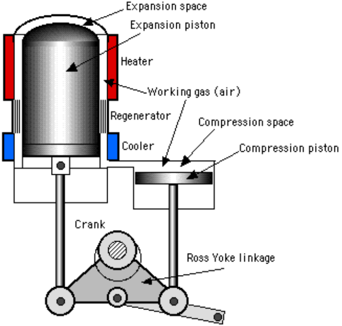
The engine

- It is an external combustion engine
- The working space is made up by
  1. An expansion space (E)
  2. A heater (H)
  3. A regenerator (R)
  4. A cooler (K)
  5. A compression space (C)
- It has two pistons
  1. A compression piston
  2. An expansion piston



from http://www.ent.ohiou.edu

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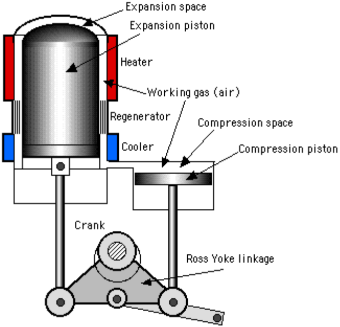

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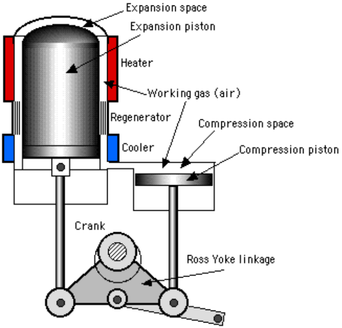

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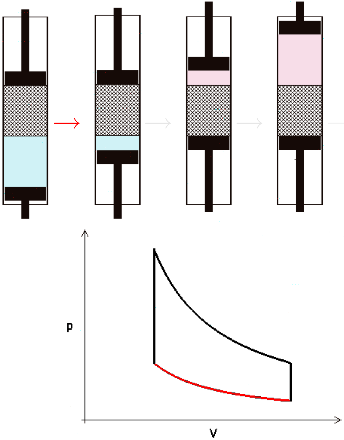
How the engine works

- The thermodynamic cycle consists in
  1. An isotherm compression
  2. An isochor displacement to the expansion space
  3. An isotherm expansion
  4. An isochor displacement to the compression space



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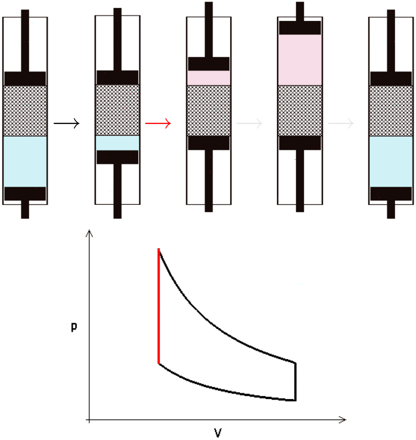

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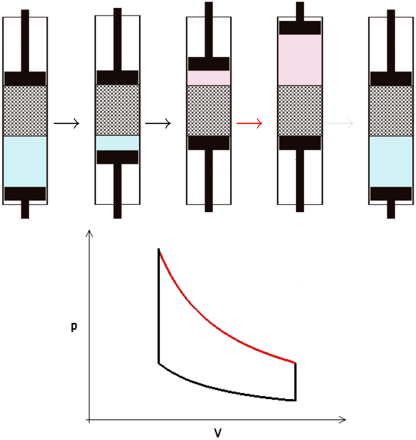

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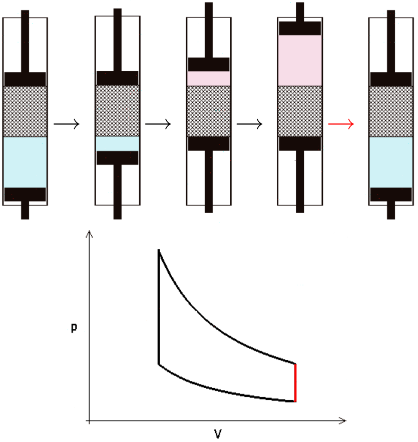

Schmidt analysis assumptions and outcome

- Main assumptions
  1. Constant fluid thermophysical properties
  2. Sinusoidal law for the expansion and the compression spaces
  3. Constant volumes for the heat exchangers
  4. Uniform pressure in the working space
  5. Constant and uniform temperature within each space
- Applying the ideal gas equation to each space and the mass conservation equation to the whole working space a closed form solution is obtained for
  1. Instantaneous pressure
  2. Instantaneous fluid masses and mass flow rates
  3. Heat exchanged and work done over a cycle
  4. Engine power output and thermodynamic efficiency

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Schmidt analysis parameters

- According to Schmidt's analysis, the following parameters fully determine the Stirling engine
  1. Expansion and compression space swept volumes
  2. Expansion and compression dead volumes
  3. Heater, regenerator and cooler volumes
  4. Heater and cooler temperatures
  5. Cycle mean pressure
  6. Phase angle between the expansion and the compression volumes
  7. Operating fluid thermodynamic constant
  8. Engine revolution speed

E		H		R		K		C	
V	T	V	T	V	T	V	T	V	T
p	M	p	M	p	M	p	M	p	M

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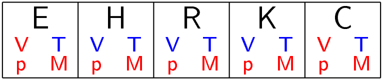

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Adiabatic analysis assumptions and outcome

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  1. Constant fluid thermophysical properties
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  3. Constant volumes for the heat exchangers
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  5. Constant and uniform temperature within each heat exchanger
  6. Adiabatic expansion and compression spaces
- Applying the ideal gas equation to each space, the mass conservation equation to the whole working space and the energy balance equation to the expansion and the compression spaces, we iterate over the whole cycle up to convergence. The solution is obtained for
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Optimization scope

- A simulation code running both the Schmidt and adiabatic analysis was written in C++
- The code was coupled with the optimization software modeFRONTIER 3.1 aiming at the maximization of both the power output and thermodynamic efficiency of the engine
- We wish to check the effectiveness of the optimization algorithms applied to the simulation of Stirling engines and to compare the differences between optimum configurations from the two models

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Constraints and variables

- We add constraints to the following parameters
  1. Cooler and heater temperature
  2. Phase angle between the expansion and the compression volumes
  3. Sum of swept and unswept volumes
  4. Cycle pressure
  5. Working fluid (helium)
  6. Revolution speed

$$T_K = 300\,\mathrm{K} \quad T_H \leq 900\,\mathrm{K}$$

$$-\pi < \alpha \leq \pi$$

$$V_{sw,E} + V_{d,E} + V_H + V_R + V_K + \\ + V_{sw,C} + V_{d,C} \leq 500\,\mathrm{cm}^3$$

$$p_{max} \leq 50\,\mathrm{bar}$$

$$R_{gas} = 2077\,\mathrm{J}/\left(\mathrm{kg\,K}\right)$$

$$c_p = 5193\,\mathrm{J}/\left(\mathrm{kg\,K}\right)$$

$$f = 600\,\mathrm{rpm}$$

- The variables of the optimization problem are
  1. The volumes of the engine
  2. The mean cycle pressure
  3. The heater temperature
  4. The phase angle between the expansion and the compression volumes

$$V_{sw,E},\ V_{d,E},\ V_H,\ V_R,\ V_K,\ V_{d,C},\ V_{sw,C},\ p_{mean},\ T_H,\ \alpha$$

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Observations

- We expect that the optimum configuration will have
  1. Maximum heater temperature
  2. Zero dead volumes and zero heat exchangers volumes
  3. Maximum swept volumes sum
  4. Maximum cycle highest pressure

$$T_H \approx 900\,\mathrm{K}$$

$$V_{d,E} + V_H + V_R + V_C + V_{d,C} \approx 0\,\mathrm{cm}^3$$

$$V_{sw,E} + V_{sw,C} \approx 500\,\mathrm{cm}^3$$

$$p_{max} \approx 50\,\mathrm{bar}$$

- If this will happen we could reduce the variables of the problem to
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Optimization steps

- The optimization procedure proceeded through three steps
    1. Several optimization techniques were applied in cascade (Sobol DOE, MOGA-II, 1P1-ES, Simplex) in order to reduce the search space size and refine the search towards the optimum configurations
    2. According to the expectations, the results from the first step asslowed us to reduced to three the number of variables. The optimization was then repeated following the same pattern (Sobol DOE, MOGA-II, 1P1-ES, Simplex) from scratch
    3. A few configurations amongst the better performing were chosen for further testing. For each of them a full factorial DOE was run varying the heater temperature from 450 K to 900 K with steps of 50 K in order to find a configuration ensuring good performance over a wide range of loads

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Schmidt analysis - first step

		Sobol	1P1-ES	Simplex
Variable		(2048)	(1024)	(1083)
		Max P	Max P	Max P
α	[deg]	123.53	110.05	111.91
T_H	[K]	839.51	888.39	899.96
$V_{sw,E}$	[cm^3]	288.62	319.86	329.92
$V_{sw,C}$	[cm^3]	114.40	175.04	169.41
$V_{d,E}$	[cm^3]	22.60	1.73	0.01
$V_{d,C}$	[cm^3]	8.58	0.00	0.23
V_h	[cm^3]	24.60	2.66	0.11
V_r	[cm^3]	21.35	0.55	0.30
V_k	[cm^3]	5.60	0.00	0.00
p_{mean}	[bar]	29.26	24.94	25.58
p_{max}	[bar]	41.69	49.70	50.00
V_{tot}	[cm^3]	485.75	499.85	499.99
P_{out}	[kW]	2.756	5.285	5.474

- We did not apply the MOGA to the Schmidt analysis since it always leads to $\eta_{stirling} = \eta_{carnot}$ making our problem strictly mono-objective

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Schmidt analysis - second and third step

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α	[deg]	115.06	112.50	113.48
$V_{sw,E}$	[cm^3]	347.88	340.97	342.84
$V_{sw,C}$	[cm^3]	152.12	159.03	157.16
p_{mean}	[bar]	26.81	26.07	26.39
p_{max}	[bar]	49.67	50.00	50.00
M_f	[g]	0.482	0.468	0.475
P_{out}	[kW]	5.469	5.512	5.513

$V_{sw,E}$	[cm^3]	350	350	340	350	350
α	[deg]	111	112	113	114	115
M_f	[g]	0.45	0.46	0.47	0.47	0.48
T_H	900 K	5.447	5.482	5.493	5.427	5.454
	850 K	5.002	5.034	5.048	4.984	5.008
	800 K	4.553	4.583	4.599	4.537	4.559
	...	...	...	...	...	...
	550 K	2.270	2.285	2.304	2.262	2.273
	450 K	1.349	1.358	1.373	1.345	1.351
Avg.	[kW]	3.410	3.432	3.449	3.398	3.414

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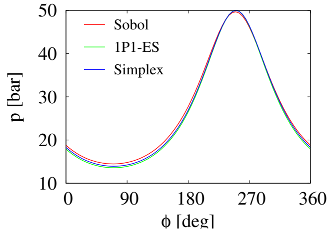
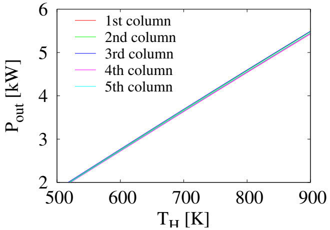
$V_{sw,E}$	[cm^3]	350	350	340	350	350
α	[deg]	111	112	113	114	115
M_f	[g]	0.45	0.46	0.47	0.47	0.48
T_H	900 K	5.447	5.482	5.493	5.427	5.454
	850 K	5.002	5.034	5.048	4.984	5.008
	800 K	4.553	4.583	4.599	4.537	4.559
	...	...	...	...	...	...
	550 K	2.270	2.285	2.304	2.262	2.273
	450 K	1.349	1.358	1.373	1.345	1.351
Avg.	[kW]	3.410	3.432	3.449	3.398	3.414





Adiabatic analysis - first step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	123.5	134.5	142.5	142.1	112.1	169.8	166.4	168.0
T_H	[K]	839.5	900.0	900.0	900.0	884.3	900.0	899.8	897.8
$V_{sw,E}$	[cm^3]	288.6	286.1	302.8	311.5	91.4	231.8	164.2	185.6
$V_{sw,C}$	[cm^3]	114.4	108.0	156.4	167.1	34.7	81.1	60.2	69.7
$V_{d,E}$	[cm^3]	22.6	20.4	21.5	0.5	15.3	23.9	22.7	20.9
$V_{d,C}$	[cm^3]	8.6	31.8	0.1	8.5	77.6	44.9	75.5	99.8
V_h	[cm^3]	24.6	32.9	9.2	10.8	37.5	0.0	38.4	15.7
V_r	[cm^3]	21.4	45.0	9.6	1.5	50.2	14.3	63.2	24.3
V_k	[cm^3]	5.6	45.0	0.1	0.1	49.0	100.0	51.5	71.5
p_{mean}	[bar]	29.3	32.7	31.5	29.8	43.7	38.8	45.5	41.6
$T_{E,max}$	[K]	864.0	921.7	942.3	949.0	892.2	903.5	903.8	902.6
$T_{E,min}$	[K]	576.6	676.7	632.0	614.8	816.5	876.0	875.3	873.8
$T_{C,max}$	[K]	428.9	394.4	417.2	428.2	324.6	308.0	307.9	307.7
$T_{C,min}$	[K]	281.7	287.9	276.5	276.7	295.4	298.6	298.2	298.0
p_{max}	[bar]	48.0	49.0	49.9	50.0	49.5	40.4	47.5	43.4
V_{tot}	[cm^3]	485.8	488.2	499.6	500.0	355.7	496.0	475.8	487.4
P_{out}	[kW]	2.61	2.86	3.87	4.05	0.40	0.36	0.30	0.30
η	[%]	50.2	57.0	54.5	53.3	63.9	66.3	66.6	66.6

Adiabatic analysis - first step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	123.5	134.5	142.5	142.1	112.1	169.8	166.4	168.0
T_H	[K]	839.5	900.0	900.0	900.0	884.3	900.0	899.8	897.8
$V_{sw,E}$	[cm^3]	288.6	286.1	302.8	311.5	91.4	231.8	164.2	185.6
$V_{sw,C}$	[cm^3]	114.4	108.0	156.4	167.1	34.7	81.1	60.2	69.7
$V_{d,E}$	[cm^3]	22.6	20.4	21.5	0.5	15.3	23.9	22.7	20.9
$V_{d,C}$	[cm^3]	8.6	31.8	0.1	8.5	77.6	44.9	75.5	99.8
V_h	[cm^3]	24.6	32.9	9.2	10.8	37.5	0.0	38.4	15.7
V_r	[cm^3]	21.4	45.0	9.6	1.5	50.2	14.3	63.2	24.3
V_k	[cm^3]	5.6	45.0	0.1	0.1	49.0	100.0	51.5	71.5
p_{mean}	[bar]	29.3	32.7	31.5	29.8	43.7	38.8	45.5	41.6
$T_{E,max}$	[K]	864.0	921.7	942.3	949.0	892.2	903.5	903.8	902.6
$T_{E,min}$	[K]	576.6	676.7	632.0	614.8	816.5	876.0	875.3	873.8
$T_{C,max}$	[K]	428.9	394.4	417.2	428.2	324.6	308.0	307.9	307.7
$T_{C,min}$	[K]	281.7	287.9	276.5	276.7	295.4	298.6	298.2	298.0
p_{max}	[bar]	48.0	49.0	49.9	50.0	49.5	40.4	47.5	43.4
V_{tot}	[cm^3]	485.8	488.2	499.6	500.0	355.7	496.0	475.8	487.4
P_{out}	[kW]	2.61	2.86	3.87	4.05	0.40	0.36	0.30	0.30
η	[%]	50.2	57.0	54.5	53.3	63.9	66.3	66.6	66.6

Adiabatic analysis - first step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	123.5	134.5	142.5	142.1	112.1	169.8	166.4	168.0
T_H	[K]	839.5	900.0	900.0	900.0	884.3	900.0	899.8	897.8
$V_{sw,E}$	[cm^3]	288.6	286.1	302.8	311.5	91.4	231.8	164.2	185.6
$V_{sw,C}$	[cm^3]	114.4	108.0	156.4	167.1	34.7	81.1	60.2	69.7
$V_{d,E}$	[cm^3]	22.6	20.4	21.5	0.5	15.3	23.9	22.7	20.9
$V_{d,C}$	[cm^3]	8.6	31.8	0.1	8.5	77.6	44.9	75.5	99.8
V_h	[cm^3]	24.6	32.9	9.2	10.8	37.5	0.0	38.4	15.7
V_r	[cm^3]	21.4	45.0	9.6	1.5	50.2	14.3	63.2	24.3
V_k	[cm^3]	5.6	45.0	0.1	0.1	49.0	100.0	51.5	71.5
p_{mean}	[bar]	29.3	32.7	31.5	29.8	43.7	38.8	45.5	41.6
$T_{E,max}$	[K]	864.0	921.7	942.3	949.0	892.2	903.5	903.8	902.6
$T_{E,min}$	[K]	576.6	676.7	632.0	614.8	816.5	876.0	875.3	873.8
$T_{C,max}$	[K]	428.9	394.4	417.2	428.2	324.6	308.0	307.9	307.7
$T_{C,min}$	[K]	281.7	287.9	276.5	276.7	295.4	298.6	298.2	298.0
p_{max}	[bar]	48.0	49.0	49.9	50.0	49.5	40.4	47.5	43.4
V_{tot}	[cm^3]	485.8	488.2	499.6	500.0	355.7	496.0	475.8	487.4
P_{out}	[kW]	2.61	2.86	3.87	4.05	0.40	0.36	0.30	0.30
η	[%]	50.2	57.0	54.5	53.3	63.9	66.3	66.6	66.6

Adiabatic analysis - first step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	123.5	134.5	142.5	142.1	112.1	169.8	166.4	168.0
T_H	[K]	839.5	900.0	900.0	900.0	884.3	900.0	899.8	897.8
$V_{sw,E}$	[cm^3]	288.6	286.1	302.8	311.5	91.4	231.8	164.2	185.6
$V_{sw,C}$	[cm^3]	114.4	108.0	156.4	167.1	34.7	81.1	60.2	69.7
$V_{d,E}$	[cm^3]	22.6	20.4	21.5	0.5	15.3	23.9	22.7	20.9
$V_{d,C}$	[cm^3]	8.6	31.8	0.1	8.5	77.6	44.9	75.5	99.8
V_h	[cm^3]	24.6	32.9	9.2	10.8	37.5	0.0	38.4	15.7
V_r	[cm^3]	21.4	45.0	9.6	1.5	50.2	14.3	63.2	24.3
V_k	[cm^3]	5.6	45.0	0.1	0.1	49.0	100.0	51.5	71.5
p_{mean}	[bar]	29.3	32.7	31.5	29.8	43.7	38.8	45.5	41.6
$T_{E,max}$	[K]	864.0	921.7	942.3	949.0	892.2	903.5	903.8	902.6
$T_{E,min}$	[K]	576.6	676.7	632.0	614.8	816.5	876.0	875.3	873.8
$T_{C,max}$	[K]	428.9	394.4	417.2	428.2	324.6	308.0	307.9	307.7
$T_{C,min}$	[K]	281.7	287.9	276.5	276.7	295.4	298.6	298.2	298.0
p_{max}	[bar]	48.0	49.0	49.9	50.0	49.5	40.4	47.5	43.4
V_{tot}	[cm^3]	485.8	488.2	499.6	500.0	355.7	496.0	475.8	487.4
P_{out}	[kW]	2.61	2.86	3.87	4.05	0.40	0.36	0.30	0.30
η	[%]	50.2	57.0	54.5	53.3	63.9	66.3	66.6	66.6

Adiabatic analysis - second step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	140.7	151.0	146.2	144.6	159.7	160.0	160.0	160.0
$V_{sw,E}$	[cm^3]	310.4	333.6	329.5	331.0	372.2	365.9	365.9	366.0
$V_{sw,C}$	[cm^3]	189.6	166.4	170.5	169.0	127.8	134.1	134.1	134.0
p_{mean}	[bar]	27.1	32.2	30.2	29.7	30.9	35.6	23.2	37.3
$T_{E,max}$	[K]	1009.2	995.0	998.9	997.7	927.2	942.6	942.5	942.2
$T_{E,min}$	[K]	569.0	650.4	616.9	608.6	725.5	730.3	730.3	730.2
$T_{C,max}$	[K]	458.1	408.1	427.9	433.2	368.9	367.0	367.0	367.0
$T_{C,min}$	[K]	204.0	129.0	230.3	19.9	289.5	276.6	276.6	276.8
p_{max}	[bar]	49.9	49.9	50.0	50.0	42.0	47.5	31.1	49.8
M_f	[g]	0.57	0.68	0.63	0.62	0.62	0.72	0.47	0.75
P_{out}	[kW]	4.15	4.19	4.27	4.27	2.72	3.23	2.11	3.38
η	[%]	50.1	55.5	53.4	52.8	59.6	59.8	59.8	59.8

Adiabatic analysis - second step

Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	140.7	151.0	146.2	144.6	159.7	160.0	160.0	160.0
$V_{sw,E}$	[cm^3]	310.4	333.6	329.5	331.0	372.2	365.9	365.9	366.0
$V_{sw,C}$	[cm^3]	189.6	166.4	170.5	169.0	127.8	134.1	134.1	134.0
p_{mean}	[bar]	27.1	32.2	30.2	29.7	30.9	35.6	23.2	37.3
$T_{E,max}$	[K]	1009.2	995.0	998.9	997.7	927.2	942.6	942.5	942.2
$T_{E,min}$	[K]	569.0	650.4	616.9	608.6	725.5	730.3	730.3	730.2
$T_{C,max}$	[K]	458.1	408.1	427.9	433.2	368.9	367.0	367.0	367.0
$T_{C,min}$	[K]	204.0	129.0	230.3	19.9	289.5	276.6	276.6	276.8
p_{max}	[bar]	49.9	49.9	50.0	50.0	42.0	47.5	31.1	49.8
M_f	[g]	0.57	0.68	0.63	0.62	0.62	0.72	0.47	0.75
P_{out}	[kW]	4.15	4.19	4.27	4.27	2.72	3.23	2.11	3.38
η	[%]	50.1	55.5	53.4	52.8	59.6	59.8	59.8	59.8

Adiabatic analysis - second step

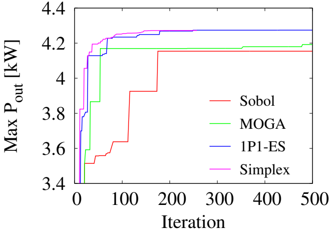
Var.		Sobol	MOGA	1P1ES	Simplex	Sobol	MOGA	1P1ES	Simplex
		Max$P$	Max$P$	Max$P$	Max$P$	Maxη	Maxη	Maxη	Maxη
α	[deg]	140.7	151.0	146.2	144.6	159.7	160.0	160.0	160.0
$V_{sw,E}$	[cm^3]	310.4	333.6	329.5	331.0	372.2	365.9	365.9	366.0
$V_{sw,C}$	[cm^3]	189.6	166.4	170.5	169.0	127.8	134.1	134.1	134.0
p_{mean}	[bar]	27.1	32.2	30.2	29.7	30.9	35.6	23.2	37.3
$T_{E,max}$	[K]	1009.2	995.0	998.9	997.7	927.2	942.6	942.5	942.2
$T_{E,min}$	[K]	569.0	650.4	616.9	608.6	725.5	730.3	730.3	730.2
$T_{C,max}$	[K]	458.1	408.1	427.9	433.2	368.9	367.0	367.0	367.0
$T_{C,min}$	[K]	204.0	129.0	230.3	19.9	289.5	276.6	276.6	276.8
p_{max}	[bar]	49.9	49.9	50.0	50.0	42.0	47.5	31.1	49.8
M_f	[g]	0.57	0.68	0.63	0.62	0.62	0.72	0.47	0.75
P_{out}	[kW]	4.15	4.19	4.27	4.27	2.72	3.23	2.11	3.38
η	[%]	50.1	55.5	53.4	52.8	59.6	59.8	59.8	59.8

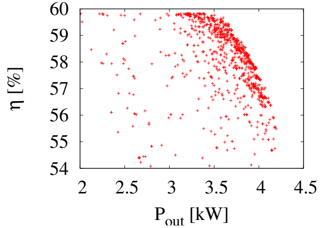
Adiabatic analysis - third step

$V_{sw,E}$	[cm^3]	330	335	335	340	345
α	[deg]	150	151	154	155	159
M_f	[g]	0.67	0.68	0.71	0.72	0.76
T_H	900 K	4.22	4.19	4.08	4.02	3.74
		54.9	55.6	56.6	57.2	58.9
	850 K	3.78	3.75	3.66	3.60	3.36
		52.6	53.2	54.4	55.0	56.8
	800 K	3.34	3.31	3.24	3.18	2.96
		49.9	50.6	51.9	52.4	54.3
	...	...	...	...	...	...
	550 K	1.12	1.09	1.11	1.07	0.99
		27.0	27.3	29.4	29.5	31.3
	450 K	0.24	0.22	0.27	0.24	0.21
		7.8	7.5	9.7	9.1	9.6
Avg.	[kW]	2.23	2.20	2.18	2.13	1.98
	[%]	37.3	37.8	39.4	39.7	41.4

Adiabatic analysis - third step

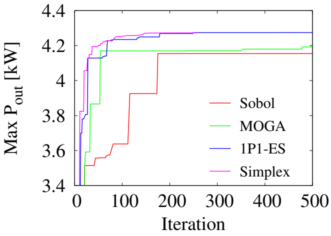
$V_{sw,E}$	[cm^3]	330	335	335	340	345
α	[deg]	150	151	154	155	159
M_f	[g]	0.67	0.68	0.71	0.72	0.76
T_H	900 K	4.22	4.19	4.08	4.02	3.74
		54.9	55.6	56.6	57.2	58.9
	850 K	3.78	3.75	3.66	3.60	3.36
		52.6	53.2	54.4	55.0	56.8
	800 K	3.34	3.31	3.24	3.18	2.96
		49.9	50.6	51.9	52.4	54.3
	...	...	...	...	...	...
	550 K	1.12	1.09	1.11	1.07	0.99
		27.0	27.3	29.4	29.5	31.3
	450 K	0.24	0.22	0.27	0.24	0.21
		7.8	7.5	9.7	9.1	9.6
Avg.	[kW]	2.23	2.20	2.18	2.13	1.98
	[%]	37.3	37.8	39.4	39.7	41.4

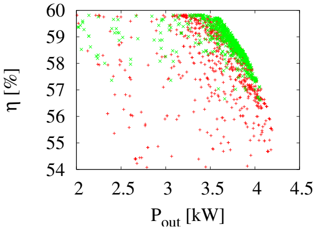




Adiabatic analysis - third step

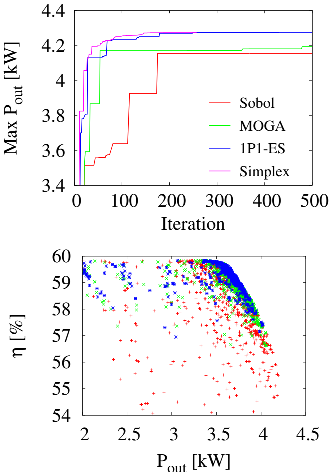
$V_{sw,E}$	[cm^3]	330	335	335	340	345
α	[deg]	150	151	154	155	159
M_f	[g]	0.67	0.68	0.71	0.72	0.76
T_H	900 K	4.22	4.19	4.08	4.02	3.74
		54.9	55.6	56.6	57.2	58.9
	850 K	3.78	3.75	3.66	3.60	3.36
		52.6	53.2	54.4	55.0	56.8
	800 K	3.34	3.31	3.24	3.18	2.96
		49.9	50.6	51.9	52.4	54.3
	...	...	...	...	...	...
	550 K	1.12	1.09	1.11	1.07	0.99
		27.0	27.3	29.4	29.5	31.3
	450 K	0.24	0.22	0.27	0.24	0.21
		7.8	7.5	9.7	9.1	9.6
Avg.	[kW]	2.23	2.20	2.18	2.13	1.98
	[%]	37.3	37.8	39.4	39.7	41.4





Adiabatic analysis - third step

$V_{sw,E}$	[cm^3]	330	335	335	340	345
α	[deg]	150	151	154	155	159
M_f	[g]	0.67	0.68	0.71	0.72	0.76
T_H	900 K	4.22	4.19	4.08	4.02	3.74
		54.9	55.6	56.6	57.2	58.9
	850 K	3.78	3.75	3.66	3.60	3.36
		52.6	53.2	54.4	55.0	56.8
	800 K	3.34	3.31	3.24	3.18	2.96
		49.9	50.6	51.9	52.4	54.3
	...	...	...	...	...	...
	550 K	1.12	1.09	1.11	1.07	0.99
		27.0	27.3	29.4	29.5	31.3
	450 K	0.24	0.22	0.27	0.24	0.21
		7.8	7.5	9.7	9.1	9.6
Avg.	[kW]	2.23	2.20	2.18	2.13	1.98
	[%]	37.3	37.8	39.4	39.7	41.4

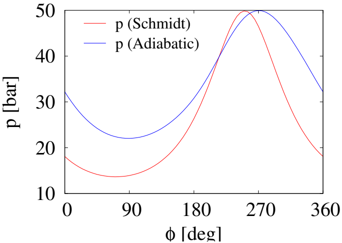


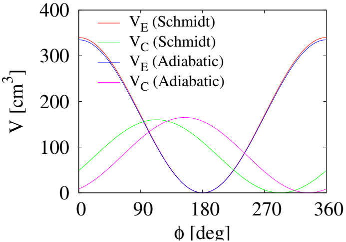
A comparison

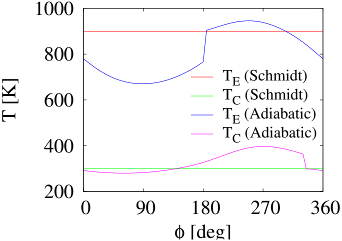
Var.		Schmidt	Adiabatic
$V_{sw,E}$	[cm^3]	340	335
$V_{sw,C}$	[cm^3]	160	165
α	[deg]	113	154
M_f	[g]	0.47	0.71
P_{max}	[kW]	5.49	4.08
P_{avg}	[kW]	3.45	2.18
η_{max}	[%]	66.6	56.6
η_{avg}	[%]	53.4	39.4

A comparison

Var.		Schmidt	Adiabatic
$V_{sw,E}$	[cm^3]	340	335
$V_{sw,C}$	[cm^3]	160	165
α	[deg]	113	154
M_f	[g]	0.47	0.71
P_{max}	[kW]	5.49	4.08
P_{avg}	[kW]	3.45	2.18
η_{max}	[%]	66.6	56.6
η_{avg}	[%]	53.4	39.4







Summary

- Though Schmidt and adiabatic analyses are far from being representative of a real machine, they are a good and very cheap starting point for investigating Stirling machines
- The algorithms employed were always effective in finding the optimum solution, in particular the Simplex method for single-objective optimization
- MOGA, though more computationally expensive than other methods, provided a good and detailed approximation of the Pareto front
- It will be of interest in the future to apply optimization algorithms to more accurate, though more computationally expensive, analyses

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